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## Symmetric–antisymmetric magnetoplasmon oscillations in a double-quantum-well structure

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**Abstract.** We predict the presence of symmetric–antisymmetric oscillations of the magnetoplasmon energy as a function of magnetic field in a double-quantum-well structure. Because electrons in the localized states cannot effectively screen the scattering potential from the impurities, in a strong magnetic field, the Landau-level width increases much faster than the  $\sqrt{B}$  dependence in a weak field. As the barrier width increases, the oscillation peaks at the smaller odd integral filling factors are smeared out before those at the larger odd integral filling factors. This leads to the quenching of the symmetric–antisymmetric magnetoplasmon oscillation peaks.

### 1. Introduction

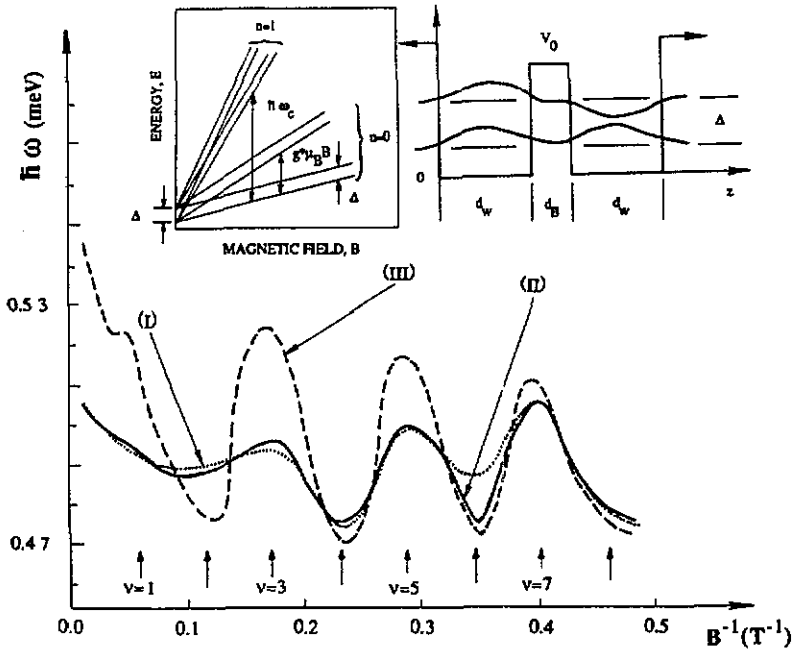
There has been great interest recently, both experimentally [1] and theoretically [2, 3], in the quantum transport behaviour of a double-quantum-well structure (DQWS). This structure introduces an additional degree of freedom in the ( $z$ ) direction perpendicular to the two-dimensional plane. In a DQWS, inter-well electron tunnelling induces a splitting of the energies for symmetric and antisymmetric states. The quantum Hall states with odd integral filling factors have been observed to be missing when the barrier width is increased [1]. This originates from the collapse of the symmetric–antisymmetric energy gap  $\Delta_{\text{SAS}}$  if the Zeeman gap is still larger than  $\Delta_{\text{SAS}}$ . In the decoupled-well regime, for large barrier separation, we have  $\nu = \frac{1}{2}\nu_{\text{tot}}$  for each well. Their observation in the DQWS amplifies the previous result for a single space-charge layer [4]. It is now well understood that the integral quantum Hall effect (IQHE) is strongly related to the localized states in the single-particle density of states sandwiched by the extended states. The existence of mobility edges arising from impurity-induced localized states is crucial to the understanding of IQHE. In [2] MacDonald *et al* have given a qualitative explanation to the observation in [1]. They argue that the Coulomb interaction will reduce the energy of the antisymmetric state, and when this reduction, scaled as  $e^2/\epsilon_0 L_{\text{H}} \propto \sqrt{B}$ , is comparable to the symmetric–antisymmetric energy gap  $\Delta_{\text{SAS}}$ , the gaps of quantum Hall states with odd integral filling factors will collapse. By using self-consistent calculations, Brey [3] has similarly predicted that the softened magnetoroton mode, associated with a large separation between the quantum wells, is responsible for the observation in [1]. However, in both [2] and [3] the impurity scattering effect is completely ignored. In the IQHE regime, the electron–electron interaction influences some features (e.g. small corrections to the spin and valley splittings), while the whole picture can be described

rather well on the basis of the effect of impurity scattering and a simple single-particle model [5]. Recently, however, there have been some experimental observations [6] in which the derivative  $\partial\mu/\partial H$  is found to be considerably higher than its maximum value for a non-interacting two-dimensional (2D) electron gas, and the thermodynamic density of states is negative near the integral filling factors. This is attributed to the electron–electron exchange interaction between quasi-particles belonging to the same Landau level. We argue that when the Landau level is completely filled, the screening of the impurity scattering potential is greatly weakened. This leads to a very large impurity-induced Landau-level broadening in a strong magnetic field. Consequently, for small  $\Delta_{\text{SAS}}$ , this disorder effect may still dominate the Coulomb interaction for a medium impurity concentration.

In this paper, on the other hand, we concentrate on the magnetoplasmon behaviour in the DQWS. We predict the existence of symmetric–antisymmetric magnetoplasmon oscillations. Although there are some studies on the oscillations of magnetoplasmons in a density-modulated 2D electron gas [7] and in a tunnelling superlattice [8], to our knowledge, there have been no reports on symmetric–antisymmetric magnetoplasmon oscillations at high magnetic fields. The basic ideas can be described as follows. Owing to the *inter-well mixing in the DQWS*, each *spin-split Landau level is further split into two levels corresponding to the symmetric and antisymmetric coupling of the Landau levels*. Therefore, a new symmetric–antisymmetric energy gap  $\Delta_{\text{SAS}}$  is formed when the magnetic field is not too small. In the presence of impurity scattering, each degenerate Landau level will be broadened into a Landau band along with the formation of mobility edges in the single-particle density of states. This allows the occurrence of symmetric inter-Landau-band, antisymmetric inter-Landau-band and coupled symmetric–antisymmetric magnetoplasmon excitations in such a system. For simplicity, we assume a maximal polarization for each state under the strong magnetic field. As the magnetic field increases, successive symmetric and antisymmetric Landau bands pass through the Fermi level and the difference of the occupation numbers for symmetric and antisymmetric states will oscillate as a function of magnetic field. This leads to strong oscillations in magnetoplasmon energy, each peak and valley corresponding to odd (symmetric–antisymmetric gap) and even (Zeeman splitting gap) integral filling factors, respectively. In some respects, this effect can be viewed as a collective analogue to the well known single-particle Shubnikov–de Haas (SDH) and de Haas–Van Alphen effects. The impurity-induced broadening is self-consistently determined by the magnetic-field dependent single-particle density of states. When the electrons are in the localized states (corresponding to integral filling factors), the screening of the impurity potential by the electron gas is greatly reduced compared with the case in which the electrons are in the extended states (half-integral filling factors). When the magnetic field is increased, we expect the electron states to be more localized, and the broadening effect will be further enhanced. In fact, in *high magnetic fields this broadening effect will be much stronger than the weak-field  $\sqrt{B}$  dependence*. The small symmetric–antisymmetric energy gap  $\Delta_{\text{SAS}}$  is believed to be magnetic-field independent. When this impurity-induced broadening becomes comparable to  $\Delta_{\text{SAS}}$  for the thicker barrier, the oscillation peaks with smaller odd integral filling factors (higher magnetic field) will be smeared out before those with larger odd integral filling factors (lower magnetic field).

## 2. Discussion

We consider a DQWS model, as shown in the inset of figure 1, with well and barrier widths  $d_{\text{W}}$  and  $d_{\text{B}}$ . The barrier height is assumed to be  $V_0$ . A strong external magnetic field is



**Figure 1.** We present here the excitation energy of the coupled symmetric-antisymmetric magnetoplasmon mode given by equation (8) as a function of  $1/B$  in a double-quantum-well structure. We can clearly see the oscillation peaks at the odd integral filling factors (curve III). As the impurity concentration increases, these peaks are largely suppressed (curves I, II). The sample parameters used for  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}-\text{GaAs}-\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  are as follows:  $m_{\parallel} = 0.079m_e$ ,  $m_{\perp} = 1.25m_0$ ,  $V_0 = 298.2$  meV,  $\epsilon_h = 10.9$ ,  $n_{2D} = 4.2 \times 10^{11}$   $\text{cm}^{-2}$ ,  $d_w = 139$  Å,  $d_B = 40$  Å,  $S = 600$  Å,  $\Delta_s = 16.32$  meV,  $\Delta_a = 16.79$  meV,  $\Delta_{\text{SAS}} = 0.47$  meV,  $\zeta_{\parallel} = 0.005$   $\text{meV}^2$ ,  $n_t D = 1.036 \times 10^9$   $\text{cm}^{-2}$  for curves I, II,  $n_t D = 2.59 \times 10^8$   $\text{cm}^{-2}$  for curve III.  $g^* = 0.65$  for curve I and  $g^* = 0.80$  for curves II, III. The arrows in the figure indicate the corresponding integral filling factors. The inset in the figure shows the single-particle energy levels as a function of magnetic field in the presence of spin splitting and inter-well mixing in a double-quantum-well structure. It also presents the symmetric and antisymmetric state wavefunctions and the potential profile in this structure.

applied along the perpendicular ( $z$ ) direction. The electron motions in the DQWs are described by the Landau quantization in the  $x$ - $y$  plane and the size quantization in the  $z$  direction. For all the samples used in [1], the second miniband, associated with the size quantization in the  $z$  direction, is largely separated from the lowest miniband, thus we only need to consider the lowest miniband. In the Landau gauge, the Hamiltonian is written as

$$H = -(\hbar^2/2m_{\parallel}) d^2/dx^2 + \frac{1}{2}m_{\parallel}\omega_C^2(x - X_0)^2 - (\hbar^2/2m_{\perp}) d^2/dz^2 + V(z) \quad (1)$$

where  $\omega_C = eB/m_{\parallel}$  is the cyclotron frequency, and  $X_0 = -\hbar k_y/m_{\parallel}\omega_C$  is the guiding centre. The DQWs potential  $V(z)$  takes the form

$$V(z) = \begin{cases} 0 & 0 \leq z \leq d_w \text{ or } d_w + d_B \leq z \leq 2d_w + d_B \\ V_0 & d_w \leq z \leq d_w + d_B \\ \infty & \text{otherwise.} \end{cases} \quad (2)$$

The eigenstates are given by

$$|k_y, N, \sigma, L\rangle = (1/\sqrt{L_y}) e^{ik_y y} U_N(x - X_0) \varphi^{(L)}(z) |\sigma\rangle \tag{3}$$

where  $L_y$  is the normalization length in the  $y$  direction.  $U_N(x - X_0)$  is the normalized wavefunction of a harmonic oscillator centred at  $X_0$ , and  $\varphi^{(L)}(z)$ , which will be given below, is the envelope function associated with the size quantization in the  $z$  direction.  $N$  is the Landau-level index, and  $L = +1, -1$  correspond to the symmetric and anti-symmetric states respectively.  $|\sigma\rangle$  ( $\sigma = +1$  for spin up and  $\sigma = -1$  for spin down) is the spin doublet. The eigenvalues are

$$E_L(N, \sigma) = (N + \frac{1}{2})\hbar\omega_c + \frac{1}{2}g^*\mu_B B\sigma + \Delta_L \tag{4}$$

where  $g^*$  is the effective Landé factor, and  $\mu_B = e\hbar/2m_c$  is the Bohr magneton.  $\Delta_{\pm}$  are the energies of the symmetric (+) or antisymmetric (-) states. Using the self-consistent-field method of Ehrenreich and Cohen [9], we get the magnetoplasmon dispersion relation [10]

$$\det \left\{ \delta_{MM':LL'} - \sum_{LL'=-1:+1} \chi_{LL'}^{(H)}(\mathbf{q}, \omega) \int dz dz' \varphi^{(M)}(z) \varphi^{(M')}(z) \right. \\ \left. \times \left[ \frac{2\pi e^2}{\epsilon_S q_{xy}} e^{-q_{xy}|z-z'|} - \frac{\delta V_{XC}[n]}{\delta n} \delta(z - z') \right] \varphi^{(L)}(z') \varphi^{(L')}(z') \right\} = 0 \tag{5}$$

where we have employed the Kohn–Sham local density approximation [11],  $\epsilon_S = 4\pi\epsilon_0\epsilon_b$  is the dielectric constant. The polarisability  $\chi_{LL'}^{(H)}(\mathbf{q}, \omega)$  is determined from

$$\chi_{LL'}^{(H)}(\mathbf{q}, \omega) = \frac{1}{2\pi L_H^2} \sum_{NN'} \sum_{\sigma} C_{NN'} \left( \frac{q_{xy} L_H^2}{2} \right) \\ \times \frac{[f_0(E_L(N, \sigma)) - f_0(E_{L'}(N', \sigma))] [E_{L'}(N', \sigma) - E_L(N, \sigma)]}{\hbar^2 \omega^2 - [E_{L'}(N', \sigma) - E_L(N, \sigma)]^2} \tag{6}$$

In equation (6),  $N$  is restricted to the occupied Landau bands while  $N'$  runs through all the Landau-level indices, and  $q_{xy}^2 = q_x^2 + q_y^2$ .  $C_{NN'}(x)$  is related to the associated Laguerre polynomials,

$$C_{NN'}(x) = e^{-x} \frac{N_2!}{N_1!} x^{N_1 - N_2} [L_{N_2}^{N_1 - N_2}(x)]^2 \tag{7}$$

with  $N_1 = \text{Max}(N, N')$  and  $N_2 = \text{Min}(N, N')$ .  $f_0[E_L(N, \sigma)]$  is the Fermi distribution function at zero temperature. In principle, equation (5) contains all the magnetoplasmon excitations of the DQWS. When  $q_{xy}$  is non-zero, in the long-wavelength limit, there exist three kinds of magnetoplasmon excitations which are coupled to each other. Two of them are symmetric inter-Landau-band and antisymmetric inter-Landau-band magnetoplasmons, which always have frequencies  $\omega > \omega_c$ . These magnetoplasmons are related to the virtual transitions between the adjacent symmetric or antisymmetric isospin Landau bands, respectively. Owing to the spin conservation for charge-density excitations, there are no transitions between different spin-related Landau bands. The other excitation is the coupled symmetric–antisymmetric magnetoplasmon mode, corresponding to the transitions between the adjacent isospin symmetric and antisymmetric Landau bands. It exhibits oscillating excitation energy as a function of magnetic field, and has the frequency  $\hbar^{-1}\Delta_{SAS} < \omega < \omega_c$  (we have neglected discussion of the spin-density excitations because they are easily Landau-damped by the single-particle–hole continuum). Since we are only interested in the impurity scattering effect on the oscillating

magnetoplasmon mode in this paper, for simplicity, we will let  $q_{xy} = 0$  (ignoring the coupling of different magnetoplasmon modes), and neglect the exchange–correlation contribution (taking  $V_{XC} = 0$ ). Under these conditions, we get the simple form for magnetoplasmon energy

$$\hbar^2 \omega^2 = \Delta_{SAS}^2 + \frac{2\pi e^2}{\epsilon_S} A \Delta_{SAS} [n^{(+)} - n^{(-)}] \tag{8}$$

where the positive depolarization-shift coefficient  $A$  can be calculated from

$$A = - \int dz dz' \varphi^{(+)}(z) \varphi^{(-)}(z) |z - z'| \varphi^{(+)}(z') \varphi^{(-)}(z'). \tag{9}$$

Assuming a Gaussian form for the single-particle density of states, we can express the occupation numbers  $n^{(\pm)}$  for symmetric and antisymmetric states as

$$n^{(\pm)} = \frac{1}{2\pi L_H^2} \int_0^{E_F} dE \frac{1}{\sqrt{2\pi\Gamma^2}} \sum_N \sum_\sigma \exp \left[ \frac{-[E - \Delta_\pm - (N + \frac{1}{2})\hbar\omega_C - \frac{1}{2}g^* \mu_B B \sigma]^2}{2\Gamma^2} \right]. \tag{10}$$

The Fermi energy  $E_F$  is determined by the total average areal electron density  $n_{2D} = n^{(+)} + n^{(-)}$ .  $\Gamma$  in equation (10) is the Landau-level width, and  $L_H = \sqrt{\hbar/eB}$  is the magnetic length. As mentioned in [12], the Landau-level width can be calculated self-consistently by using a simple two-dimensional Thomas–Fermi model

$$\Gamma^2 = \zeta_L x / (4 + x) \quad x^2 = 2L^2 / L_H^2 \tag{10a}$$

$$\zeta_L = (n_1 D / \pi L^2 q_S^2) (2\pi e^2 / \epsilon_S)^2 + \zeta_0 \tag{10b}$$

$$L^2 = \sqrt{S^2 + 1/Q_S^2} \tag{10c}$$

$$q_S^2 = (2\pi e^2 / \epsilon_S) n^{2D}(E_F) \quad Q_S^2 = (2\pi e^2 / d_w \epsilon_S) n^{2D}(E_F) \tag{10d}$$

$$n^{2D}(E) = \frac{1}{2\pi L_H^2} \frac{1}{\sqrt{2\pi\Gamma^2}} \sum_L \sum_N \sum_\sigma \exp \left[ \frac{-[E - \Delta_L - (N + \frac{1}{2})\hbar\omega_C - \frac{1}{2}g^* \mu_B B \sigma]^2}{2\Gamma^2} \right] \tag{10e}$$

$$n_{2D} = \int_0^{E_F} n^{2D}(E) dE. \tag{10f}$$

In equations (10)  $\zeta_0$  is a constant value which is added to include the other broadening effects (e.g. spatial inhomogeneities, electron–phonon scattering). The  $\zeta_0$  term also helps to stabilize the iterative solution by preventing  $\zeta_L$  from vanishing during the iterations.  $S$  is the spacer layer thickness,  $L$  is the correlation length,  $n_1$  is the impurity concentration and  $D$  is the thickness of the doping layer. The main model parameter  $n_1 D$  here can be approximately estimated from the zero-field mobility value. Assuming

a zero-range impurity scattering potential, in the absence of magnetic field we get for the lowest self-consistent Born approximation [13]

$$\hbar/\tau = 2\pi|V_1|^2 n_1 D n_0^{2D}(E_F) \quad (11)$$

where the scattering time  $\tau$  is related to the mobility  $\mu$  by

$$\hbar/\tau = \hbar e/m_{\parallel}\mu. \quad (12)$$

Making use of the impurity-impurity potential correlation function, we can express the scattering potential amplitude by the correlation length and correlation strength

$$|V_1|^2 = \xi_L(\pi L^2)^2. \quad (13)$$

Taking  $L = S$  (wide spacer approximation) and  $n_0^{2D}(E_F) = m_{\parallel}/\pi\hbar^2$  (for zero magnetic field), and substituting equations (12) and (13) into equation (11), we finally get

$$n_1 D \sim \sqrt{e^2/2\pi^3 S^2 \mu \hbar}. \quad (14)$$

Substituting the experimental parameters given in [1] for this  $d_B = 40 \text{ \AA}$  case, we estimate  $n_1 D \sim 1.036 \times 10^9 \text{ cm}^{-2}$ . For the calculation of depolarization-shift coefficient  $A$ , we write the envelope function  $\varphi^{(\pm)}(z)$  using first-order perturbation theory

$$\varphi^{(\pm)}(z) = \begin{cases} (1/\sqrt{d_W}) \sin(k_{\pm} z) & 0 < z \leq d_W \\ (1/\sqrt{d_W})(k^{(0)}/\kappa^{(0)}) [e^{-\kappa^{(0)}(z-d_W)} \pm e^{-\kappa^{(0)}(d_W+d_B-z)}] & d_W < z \leq d_W + d_B \\ (\pm 1/\sqrt{d_W}) \sin(k_{\pm}(2d_W + d_B - z)) & d_W + d_B < z \leq 2d_W + d_B \end{cases} \quad (15)$$

where

$$k_{\pm} = k^{(0)} - (k^{(0)}/d_W \kappa^{(0)}) \mp 2(k^{(0)}/d_W \kappa^{(0)}) e^{-\kappa^{(0)} d_B} \quad (15a)$$

$$k^{(0)} = \pi/d_W \quad \kappa^{(0)} = \sqrt{2m_{\perp}(V_0 - E_1^{(0)})/\hbar^2} \quad E_1^{(0)} = \pi^2 \hbar^2 / 2m_{\perp} d_W^2 \quad (15b)$$

$$\Delta_{\pm} = E_1^{(0)} [1 - 2/d_W \kappa^{(0)} \mp (4/d_W \kappa^{(0)}) e^{-\kappa^{(0)} d_B}] \quad (15c)$$

$$\Delta_{SAS} = (8E_1^{(0)}/d_W \kappa^{(0)}) e^{-\kappa^{(0)} d_B}. \quad (15d)$$

From the experimental value  $\Delta_{SAS} = 0.47 \text{ meV}$ , we get  $m_{\perp} = 1.25 m_{\parallel}$ ,  $\Delta_+ = 16.32 \text{ meV}$ , and  $\Delta_- = 16.79 \text{ meV}$ . The oscillation period (in  $1/B$ ) is  $\Delta_{SAS}(1/B) = 4\pi e/\hbar n_{2D}$  which is different from the period  $\Delta(1/B) = e\hbar/m_{\parallel} E_F$  for single-particle Shubnikov-de Haas or de Haas-Van Alphen effects. It is evident from equation (8) that the oscillation peak is proportional to the symmetric-antisymmetric energy gap  $\Delta_{SAS}$ , the depolarization effect, and the difference of occupation numbers for symmetric and antisymmetric states. As the barrier width increases, the inter-well electron tunnelling is decreased (reducing  $\Delta_{SAS}$ ), and the oscillation peaks at odd integral filling factors will be largely suppressed. In this case the filling factor  $\nu = 2\pi L^2_{\parallel} n_{2D}$  would represent the filling factor  $\nu/2$  in each independent single quantum well ( $n_{2D}/2$  is the average areal electron density in each single quantum well). This implies that there are generally no oscillations of magnetoplasmon excitation energy in each single quantum well. Under the assumption  $q_{xv} = 0$ , only the cyclotron mode  $\omega = \omega_C$  exists in each single quantum well. For a lower impurity concentration  $n_1 D \sim 2.59 \times 10^8 \text{ cm}^{-2}$  (curve III), which is nearly an order of magnitude lower than that of the sample ( $d_B = 40 \text{ \AA}$ ) used in [1], the magnetoplasmon excitation energy shows large oscillation peaks at the odd integral filling factors except

at  $\nu = 1$  where the Landau-level width  $\Gamma$  increases much faster than the weak-field  $\sqrt{B}$  behaviour. For medium impurity concentration  $n_1 D \sim 1.036 \times 10^9 \text{ cm}^{-2}$  (curve II), like that in the sample ( $d_B = 40 \text{ \AA}$ ), the original small peak at  $\nu = 1$  is completely smeared out, and the peaks at  $\nu = 3, 5$  are greatly suppressed (but still visible). It is evident that the strength of peaks is very sensitive to the impurity concentration. This indicates that impurity scattering may play an important role here. The smaller the odd integral filling factor is, the weaker the oscillation peak is. On the other hand, the valleys, associated with the even integral filling factors, are almost uncharged except at  $\nu = 2$  where the larger Landau-level width  $\Gamma$  at high magnetic field is comparable to the Zeeman splitting. The effect of Zeeman splitting can be clearly seen with the decrease of  $g^*$  from 0.80 (curve II) to 0.65 (curve I), where the depth of the valley at  $\nu = 6$  for low magnetic field is greatly reduced. We emphasize that for a fixed impurity concentration (i.e. the Landau-level width fixed at a definite magnetic field) the change of the magnetoplasmon energy in the spectrum with an increase of barrier width (reducing  $\Delta_{\text{SAS}}$ ) can be equally well explained by increasing the impurity concentration.

### 3. Conclusion

The main component of this paper is the prediction of the symmetric–antisymmetric magnetoplasmon oscillations in a DQWS. This is the first time that the magnetoplasmon oscillations in a double-quantum-well structure have been reported. This effect can be used to measure, for example, the Landau-level width  $\Gamma$ , the energy gap between two Landau bands, the effective Landé factor  $g^*$ , the inter-well electron tunnelling, the Hartree-type Coulomb interaction between electrons, and even the single-particle density of states. Light-scattering measurements of this interesting collective phenomenon can be further compared with those of the single-particle Shubnikov–de Haas and de Haas–Van Alphen effects. The advantage of a DQWS is the control of the inter-well electron tunnelling or the symmetric–antisymmetric energy gap  $\Delta_{\text{SAS}}$  by changing the barrier width. It also allows an accessible range of magnetic field for emptying the Landau bands. This leads to the direct modulation of the magnetoplasmon oscillation peaks. Since, at present, we are mainly concerned with the effect of impurity scattering on the magnetoplasmon excitations, we only include the Hartree-type Coulomb interaction and the discussion of the special case  $q_{xy} = 0$  where there is no coupling between different magnetoplasmon modes. Also, valley splitting and spin-density excitations, associated with the transitions between different spin states, are not included, and the theory is restricted to the zero-temperature case. In the self-consistent calculation of the Landau-level width, we have employed a simple 2D Thomas–Fermi model. We believe that the impurity disorder should still play an important role in the DQWS since the majority of electrons in these samples are introduced by doping. Although the spacer layer thickness can be large, the electrons in the localized states (at integral filling factors) are still unable to screen the impurity scattering potential effectively.

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